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WASHINGTON DEPARTMENT OF NATURAL RESOURCES

PUBLIC LAND SURVEY OFFICE AND SURVEY ADVISORY BOARD

SURVEYOR'S GUIDEBOOK ON

RELATIVE ACCURACY

COVER

ACKNOWLEDGMENTS

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June 1995

Dear Reader:

This "Relative Accuracy Guidebook" has been made possible by an incredible effort of the Relative Accuracy Committee, a diverse group of public and private land surveyors. The Committee consists of the Survey Advisory Board and representatives from the Land Surveyors' Association of Washington.

This guidebook was prepared pursuant to the authority of RCW 58.24, which requires our agency to cooperate and consult with state, county and municipal governments and registered land surveyors for the establishment of survey standards, methods of procedure, and the monumentation of boundaries.

The Relative Accuracy Committee and DNR staff have put many hours of hard work and dedication into this project. Please give serious consideration to the information presented in this guidebook as the DNR and the surveying community work cooperatively for the benefit of the people and the industry of our state.

Sincerely,

JENNIFER M. BELCHER
Commissioner of Public Lands

June 1995

The Relative Accuracy Committee is pleased to present this guidebook and introduce the surveying community to the relative accuracy subject.

We do not intend this guidebook as a complete discussion of the subject of relative accuracy. It is an introduction that hopefully will encourage the reader to pursue additional study. The committee members strongly believe that it is in the best interests of the surveying profession, and the public served by this profession, to incorporate this concept into everyday practice.

The initial impetus for this guidebook came from discussions occurring during the recent revision of the Survey Standards. In addition, changes in surveying equipment and procedures have shown the need for a reevaluation of current practices for analyzing survey work. Communications with representatives from other states and from the Land Surveyors' Association of Washington emphasized the need for a cooperative learning process to make such a transition successful.

Therefore, the committee views this guidebook as a first step in this interactive, educational effort. As such it was intentionally left unfinished at this time with the idea being that review by the surveying community will help "flesh out" the remaining text. We encourage your active involvement in this process. Please address any comments you may have about this guidebook or the subject of relative accuracy to:

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The committee will greatly appreciate your participation.

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SURVEYOR'S GUIDEBOOK ON

RELATIVE ACCURACY

Preface

It is in the best interests of the surveying profession for surveyors and their clients to know the relative accuracy with which the locations of property corners have been identified. The client and adjoiners have more interest in the correctness of the location of the points than in the level of precision with which they were tied together. High precision does not imply good accuracy; accuracy involves much more. The ability to measure with great precision does not remove the need for thorough surveying procedures.

For example, a professional land surveyor is delighted with the results obtained using a new 1″-theodolite and EDM. The 1:61,000 closure for the first survey performed with the new equipment may have been a fluke, but 1:72,163 for the second survey is very satisfying indeed. For years the surveyor had struggled with tape and transit, rarely achieving precise results that were even close to those in this new "string of successes".

However, on the second survey, even though an excellent closure is achieved and the computed property lines match the record bearings and distances, there is a problem with the north and east property lines. A recently established rebar found near the surveyor's calculated position for the northeast corner appears to be the property corner, but is 0.39 ft southwest of the position. Additionally, there is a recently placed iron pipe about 1 ft east of the surveyor's calculated location.

Considering the closure, the surveyor is inclined to set a new monument. Certainly, the pipe could be rejected, and in addition, the 0.39 ft discrepancy could be significant to the surveyor's client. Yet, the records show that all the surveys used the same controlling monuments. Is there some guide that can be used to logically decide how close is close enough before reestablishing a property corner near an existing monument?

Because of the increased use of radial surveys, analytical photogrammetry, Global Positioning System technology, and other techniques, surveyors now need a better method for analyzing traverses than linear error closure used for many years.

This book is a guide for making those analytical decisions. It is intended as a tool for enhancing professional judgements.

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1. INTRODUCTION

1. Introduction

The professional land surveyor is responsible for selecting and using proper equipment, procedures, and analytical methods to achieve appropriate survey accuracy. To meet this responsibility the surveyor, before beginning a survey, estimates the accuracy expected and, after completion, computes the accuracy that has been obtained.

WAC 332-130-070 states "The accuracy or precision of field work may be determined and reported by either relative accuracy procedures or field traverse standards, provided that the final result shall meet or exceed the standards contained in WAC 332-130-090." **The purposes of this guidebook are to identify the basic principles of relative accuracy and to guide surveyors toward analysis of their work by these procedures.** This guidebook is a practical introduction to analyzing the certainty of survey measurements and is not intended as a complete or theoretical discussion.

An analysis of survey measurements before and after a survey provides a basis for accuracy evaluation. From this knowledge the surveyor is better able to:

- Select suitable instrumentation and appropriate measurement procedures.
- Formulate an economic basis for survey designs.
- Properly locate and preserve the position of property corners.
- Show compliance with a standard.
- Achieve higher accuracy when needed.

The surveyor is an expert in analyzing survey measurements. This is what separates the professional from the technician.

Evidence and Procedures for Boundary Location by Brown and Eldridge, 1962, states on page 441:

> "The surveyor should never be so foolish as to represent his measurement as being exact and without error, for a wise attorney, upon cross examination, will quickly discredit such testimony. A frank statement of expected error and the sources of errors must be presented..."

2. GENERAL CONCEPTS

2. General Concepts

A major part of surveying consists of combining the art and science of measuring to determine the relative positions of existing points, to establish new points, and to reestablish lost and obliterated points in specified positions. This section discusses general concepts of the science of measurements. (For an indepth study of this subject the reader is referred to the Selected References.)

2.1 Errors and Mistakes

The term error describes the difference between an observed or computed value of a quantity and the true value of that quantity. Errors are generally classified into two basic categories, systematic and random.

Mistakes are not to be confused with errors. Mistakes are blunders and are normally caused by incorrect readings or defective equipment.

<u>Systematic Errors</u>

A systematic error follows a pattern that will be duplicated if the measurement or calculation is repeated under the same conditions. Some examples of systematic errors are:

- Measuring with a tape that is too short or too long.
- The personal bias of an observer.
- Not correcting for the prism offset.
- Applying the grid factor incorrectly to all distances.
- Using an unadjusted instrument.

The surveyor should use procedures to detect and correct all possible systematic errors. If systematic errors are reduced so that they are negligible, then measures of precision can be used as measures of accuracy.

<u>Random Errors</u>

Random errors are caused by instrument reading errors, imperfection in instrument design, instrument wear, and unpredictable natural forces. They are arbitrary in nature, and the sign and magnitude are never known. Although the magnitude can be reduced by using care and proper procedures, these errors can never be eliminated. Random errors follow the laws of probability and tend to compensate, but never completely.

Examples of random errors are:

- Pointing instruments.
- Reading instruments.
- Setting sights, prisms and instruments.

- Marking or setting points.

2.2 Precision and Accuracy

Precision reflects the degree of refinement in performing an operation and in the quality and adjustment of the equipment. The closer the agreement of a series of readings, the more precise the value will be. Precision and random errors are closely related. If random errors are small, precision should be high, provided no mistakes have been made.

Accuracy is the degree of conformity or closeness of a measurement to the true value and indicates the extent of attention given to detecting and removing systematic errors and mistakes. Accuracy relates to the true quality of a result and is distinguished from precision, which relates to the quality of the operation.

2.3 Significant Figures

Because no measurement is exact, the surveyor must properly represent the significant figures in both reported measurements and computed results. The definition of significant figures from R. B. Buckner's 1983 text, *Surveying Measurements and their Analysis* (p.84) seems appropriate.

> "The number of significant figures in any measured quantity is the number that have meaning as estimated from the order of precision with which the quantity was measured, said meaning being limited by the judgment of the person evaluating the observation and the precision."

The accuracy of a reported value is considered to be plus (+) or minus (-) one-half the last place. Let's say the distance between found monuments in a 30-point traverse was calculated to be 5291.84 ft, with an estimated uncertainty of ±0.5 ft. Showing this distance on a map as 5291.84 ft, in the absence of a qualifying statement of uncertainty, means that it is significant to ±0.005 ft. This implies that the actual distance would be between 5291.835 and 5291.845, which, in turn, implies an accuracy of approximately 1:1,000,000. This is not true. Therefore, an appropriate qualifying statement of accuracy would be 5291.8±0.5 ft which denotes a more realistic accuracy of approximately 1:10,000.

A reported direction to the nearest second implies an accuracy of 1:206,200. A 1″ directional error in 5280.00 ft would give a perpendicular distance error of 0.025 ft to the direction of the line. Using the above 30-point traverse example with an uncertainty of ±0.5 ft, the direction would have an uncertainty of ±20″. In this case the angular accuracy would be the same as the distance accuracy (i.e., 1:10,000). Uncertainties in distance and direction could be compatible but are not necessarily measured to the same precision.

2.4 Mean, Standard Deviation, and Standard Error of the Mean

The best estimate of a quantity is the average of the measurements of that quantity. The standard deviation is a statistic that describes the range of measured values. Due to the large number of measurements required to quantify a length or angle, surveyors will typically approximate the average or mean μ by $\bar{x}$, the estimate of the mean. Similarly, the standard deviation σ is usually approximated by s_x, the estimate of standard deviation.

The Gaussian, normal, or bell-shaped distribution shown in Figure 2.4.1 is the most common probability density function used in describing surveying measurements. Most such observations follow random error theory, small errors are more likely than large errors and positive errors are as likely as negative errors. Any single measurement, selected at random from an entire set of observations, will fall within (i.e., plus or minus) one standard deviation of the mean for 68.3% of all observations. The true value, μ, as shown in the figure is never really known and could fall anywhere under the bell-shaped curve.

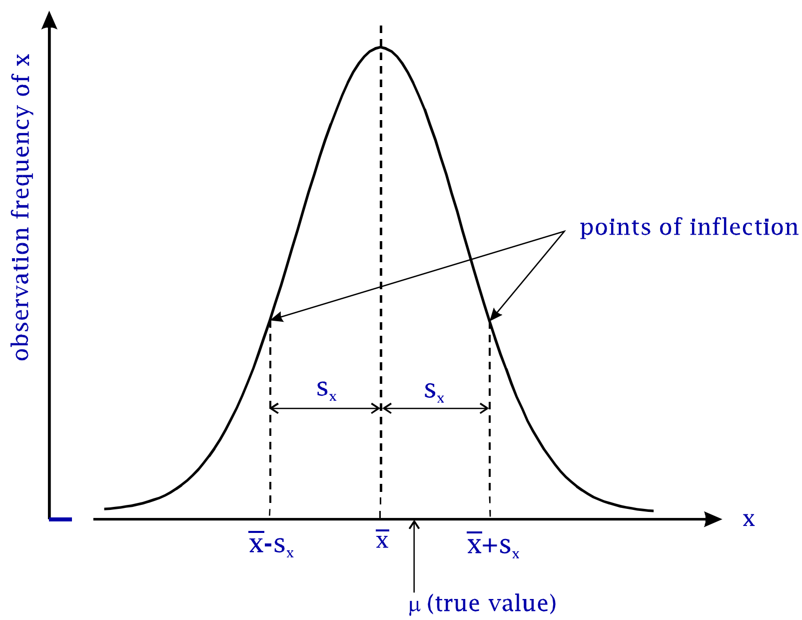

FIGURE 2.4.1. A Gaussian or normal probability distribution with estimated mean and standard deviation.

Figure 2.4.2 demonstrates the inverse relation between standard deviation and precision. A low standard deviation correlates with high precision.

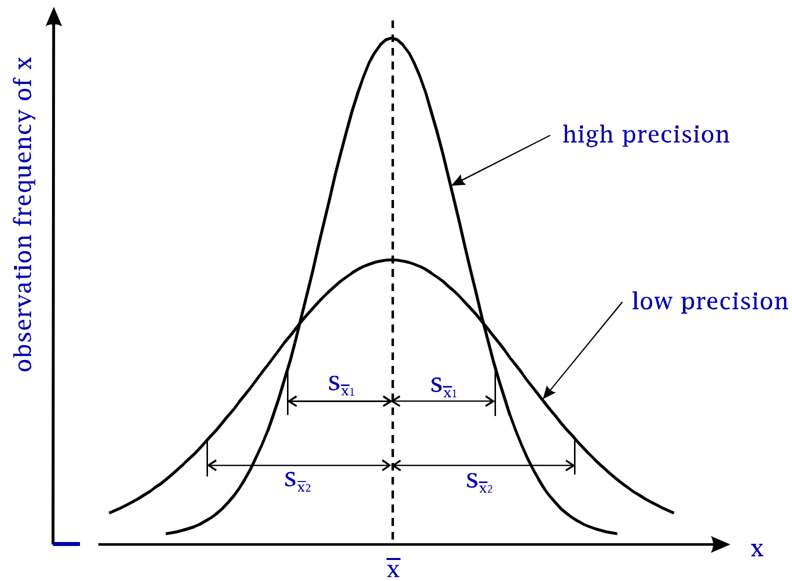

FIGURE 2.4.2. Comparison of normal distributions for measurements using high precision and low precision.

The equation for estimating standard deviation, s_x is:

$$s_x = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} \qquad \text{Eq.1}$$

where:
n = the number of independent observations within a set of measurements.
x = value obtained by independent observation or measurement.
$\bar{x}$ = arithmetic mean (average), which is equal to the sum of the independent observations or measurements, divided by n.
$(x-\bar{x})$ = the differences between each of the independent observations and the mean, called the residuals.
$\sum(x-\bar{x})^2 = (x_1-\bar{x})^2+(x_2-\bar{x})^2+\cdots+(x_n-\bar{x})^2$, called the sum of the squares of the residual.

(For a more indepth explanation of the relationship of $\bar{x}$ to μ and s_x to σ, see Table 1)

Equation (1) results in only an approximation unless there is an infinite number of independent observations. An independent observation consists of setting up the instrument, backsighting and foresighting, reading the angles, and measuring the distance. The number of independent observations that must be made for a sample test depends on how many are needed for the observer to gain a level of confidence that any errors are primarily random in nature. This is a matter of professional judgment, although some texts suggest at least 15 to 25 independent observations. To better understand the practical application of determining the procedures to use and the number of independent observations to be made, see Section 9 of A. Barry's 1978 text, *Errors in Practical Measurements in Science, Engineering, and Technology*.

The standard error of the mean is an uncertainty statement regarding the mean of a set of independent observations. Systematic errors and mistakes must be corrected for, so the mean can approach the true value with the probability of the standard deviation or a multiple of it (i.e., 1.64*s*, 90% probability; 1.96*s*, 95% probability). The standard error of the mean is extremely useful for designing measuring systems because it helps the surveyor select the appropriate equipment and number of observations needed to achieve a particular desired resultant.

The equation for the standard error of the mean, $s_{\bar{x}}$, is:

$$s_{\bar{x}} = \pm\sqrt{\frac{\Sigma(x-\bar{x})^2}{n(n-1)}} = \pm\frac{s_x}{\sqrt{n}} \qquad \text{Eq.2}$$

where all terms and symbols are as defined previously. Note that $s_{\bar{x}}$, the standard error of the mean, has a smaller value than s_x, the standard deviation of the set of independent observations. In other words, the mean of a set of observations should be closer to the true value than one independent observation and would be reflected in a smaller standard deviation value.

2.5 Confidence Level or Level of Certainty

Confidence level or level of certainty numerically states the level or degree of confidence attributed to a particular measurement through analysis of errors. As with the application of any concept, professional judgment will be needed.

The statistic that should be used in calculating the range of acceptable values for a given measurement at a particular confidence level is called "Student's t", or simply the *t* statistic. Table 1 lists theoretically derived *t* values. Note that the *t* values depend not only on the level of confidence but also on the degrees of freedom. Statistical degrees of freedom for multiple measurements of a quantity are established by the number of times the measurement is made. Each observation after the first one is called a redundant measurement. Any measurements beyond that single measurement increase the reliability of the result and simultaneously increase the degrees of freedom.

For example, the angle between two lines can be established by a single measurement. An angle measured *n* or 4 times will produce *n*-1 or three degrees of freedom for use in establishing the range of acceptable values for the angle. One degree of freedom has been removed from the total of *n* or 4 measurements to estimate the average or mean value for angle measurement.

Example Problem 2.5.1:

A line has been measured twice, resulting in values of 345.67 and 345.65 ft. With 95% confidence, what is the estimated length of the line? (This example is simplified and includes only random errors in observation.)

First, calculating the mean of the set of observations, where x_1 = 345.67 and x_2 = 345.65:

$$\bar{x} = \frac{\Sigma x}{n} = \frac{x_1 + x_2}{2} = \frac{345.67 + 345.67}{2} = 345.66$$

The standard deviation of the set of observations is estimated by:

$$s_x = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2}{(n-1)}} = \sqrt{\frac{(0.01^2) + (-0.01^2)}{(2-1)}} = 0.014$$

The standard error of the mean of the set of observations is estimated by:

$$s_{\bar{x}} = \pm\frac{s_x}{\sqrt{n}} = \pm\frac{0.014}{\sqrt{2}} = \pm 0.01$$

To be assured with 95% confidence that the mean is correct, the range of expected mean values must also be determined statistically. The degrees of freedom are needed to estimate this range of values. In this example the two measurements produce n-1 or one degree of freedom. Using one degree of freedom and a 95% confidence level, the value for t is 12.7 as obtained from Table 1. The resulting 95% confidence interval is:

$$\bar{x} \pm (s_{\bar{x}})(t) = 345.66 \pm (0.01)(12.7) = 345.66 \pm 0.13$$

The range of acceptable values with 95% confidence is 345.66±0.13 or between 345.53 and 345.79.

Considering the precision with which the line was measured (345.67 and 345.65), most surveyors would be more certain about the length of the line than this analysis would reveal. But in believing that a higher certainty is appropriate, additional information and experience are inherently assumed. Many would think: "I've used the same procedure in measuring this line as I have in hundreds or thousands of similar lines". If this "experience" can be assumed to be directly applicable to the measurement of this line, the number of degrees of freedom used in obtaining a t value will increase tremendously, even approaching an infinite number of degrees of freedom, resulting in a t value of 1.96. Statisticians refer to such information as a reference distribution.

The resulting range of values containing 95% of the estimates of the mean might approach:

$$\bar{x} \pm (s_{\bar{x}})(t) = 345.66 \pm (0.01)(1.96) = 345.66 \pm 0.02$$

or between 345.64 and 345.68. We caution surveyors that each measurement should be performed independently. To consider this approach as valid, certain professional decisions are necessary regarding the number of times to measure an angle, length, or elevation and how to decide the number of measurements that can be replaced by "experience".

What level of confidence or level of certainty should be used for a measurement or established position of a corner? Each confidence level has a confidence coefficient. The most commonly used are 1*s* for 68.27% confidence, 1.64*s* for 90%, 1.96*s* for 95%, and 3*s* for a 99.7% confidence level. Although higher levels of confidence may be appropriate for certain applications, we recommend 2*s* (95.5% probability). This is consistent with the *Standards and Specifications for Geodetic Control Networks*, published by the Federal Geodetic Control Committee in 1984.

2.6 Propagation of Random Error

Each set of *n* observations results in a single mean quantity $\bar{x}$, but it may consist of many operations, each of which has random errors. The mean value of an angle measured by repetition is derived from the process of setting two targets, setting up a theodolite, pointing the instrument multiple times, and reading the corresponding values. While pointing and reading the instrument can contribute to the observation error for the angle, setting the targets and theodolite have random errors independent of the observed angle. The measurement of distances also consists of many operations, each of which can have independent random errors.

The mathematical determination of the sum of these errors as they accumulate, cancel, decrease, or otherwise behave is termed the propagation of random errors. The general formula, shown below, is simply the square root of the sum of the squares of the individual errors.

$$E_{s_x} = \sqrt{s_{x_1}^{\ 2} + s_{x_2}^{\ 2} + s_{x_3}^{\ 2} + \cdots + s_{x_n}^{\ 2}} \qquad \text{Eq.3}$$

where s_{x_1}, s_{x_2}, etc., are random errors of any independent observation or measurement that would affect the final compiled value.

The formula for the propagation of random error in a series is:

$$E_{s_x} = s_x\sqrt{n} \qquad \text{Eq.4}$$

where:

s_x = random error of any independent observation or measurement that occurs more than once in a measurement system.

n = number of times the above random error has an opportunity to occur.

The six general concepts of the science of measurements should provide you with the foundation to further study the analysis of survey measurements.

3. ANALYSIS OF SURVEY MEASUREMENTS

3. Analysis of Survey Measurements

In the previous chapters we described error analysis in broad terms. Now we need to be more specific and look at the actual errors that can occur in measurement systems (i.e., distance, direction, and the resulting position). The following discussion and examples provide some basic concepts.

Example:

A surveyor has completed all field work and office computations for a resurvey of a one-acre lot in a previously filed plat. The purpose of the survey was to find or reestablish all lot corners and stake the east line so the client can build a garage with the appropriate setback.

All controlling block corners were found; they were marked with the original monuments. The only original lot corner found was the northwest corner. The northeast and southeast lot corners were monumented by a private surveyor during a resurvey of the adjoining lot. A record of this survey had been filed at the county auditor's office. However, this survey map did not identify what controlling corners, if any, were used to set the lot corners. Both surveys determined the original corners on this east line had been lost. Ties to the previous surveyor's corners indicate an overlap of 0.23 ft on the east line.

The surveyor, knowing that the overlap and the found corners will have to be shown, wonders which survey is more accurate.

3.1 Errors in Distance

The error in measuring any distance depends on many factors. These errors may be systematic and/or random. Systematic errors should be identified and eliminated, or corrections should be applied to reduce them to an insignificant value.

Taped Measurements - The following primary sources of taping errors are all systematic errors:

- Calibration
- Temperature
- Tension
- Slope
- Sag
- Alignment

Reading precision and other procedures can cause random errors in the determination of each of these systematic errors.

The following primary sources of taping errors are all random errors:

- Reading the temperature.
- Reading the tension.
- Plumbing and reading the tape.
- Marking a point.
- Reading the vertical angle.

These must be computed according to the rules of random error propagation to determine their effect on surveying measurements.

Example Problem 3.1.1:

A 200-ft calibrated steel tape has been used to measure the distance between two block corner monuments, A and B. The measurement was done in four segments by setting three points, each at a distance of 200.00 ft, then measuring the last segment to monument B. The sum of the four measurements is 779.86 ft. The tape was standardized, unsupported, at 65°F with 25 lb tension, and calibrated at 200.00 ft under those conditions. The measurement between A and B was made with the tape unsupported, with plumb bobs, but without a tension handle. From practical observations and testing results on the procedures used by your survey crew you know that:

- Tension is within ±5 lb.
- Temperature reading is within ±10°F.
- Alignment error is within ±0.5 ft.
- Plumbing and reading of the tape is within ±0.01 ft.
- Marking a point is within ±0.01 ft.
- Vertical angle is within ±2′.

The following data are from your field book:

From monument A:
Segment 1: 200.00 ft; vertical ∠: -1°56′; set intermediate point.
Segment 2: 200.00 ft; vertical ∠: -4°09′; set intermediate point.
Segment 3: 200.00 ft; vertical ∠: -3°22′; set intermediate point.
Segment 4: 179.86 ft; vertical ∠: -13°41′; to monument B.
Temperature = 80°F. Tension = 25 lb.

What is the corrected horizontal distance between monuments A and B, and with what certainty should the distance be reported?

The corrected horizontal distance is determined by making adjustments for vertical angle and temperature difference. Applying these corrections to the measured slope distance yields a horizontal distance between monuments A and B of 773.85 ft.

The computation of the uncertainty of the computed horizontal distance is more involved. Each individual source of error can be isolated and computed for its total effect by taking into consideration the repetition within the measurement system.

For example, the plumbing and reading of the tape is repeated twice for each of the four taping segments, which totals eight repetitions of plumbing and reading errors. Using the equation for the propagation of random error in a series, Eq.4, the total expected error for this procedure is equal to ±0.028 ft (see page 3-5 for the calculations).

The surveyor must determine whether each source of error will be derived independently from each measurement task or can be distributed throughout the length of the distance taped. The surveyor must also note whether the errors compensate according to the law of compensation or accumulate directly with the number of observations. Errors in plumbing and readings, applying tension, and marking of points accumulate according to the law of compensation. Temperature errors may compensate or accumulate depending on whether the temperature is taken as a daily average or at the time of each measurement. Alignment errors, which randomly fall on either side of the correct alignment, accumulate directly with the number of observations and always make the measurement longer.

But, to keep the calculations within perspective, let's go through the process systematically.

- Tension errors - Applying a tension that is inconsistent with the standardization of the tape will result in measurement errors due to both the expansion and sag characteristics of a tape.

  Expansion: Using averages for the dimensions and modulus of elasticity of steel tapes, the tension error related to expansion, for the total length, is ±0.011 ft.

$$E_e = \left(\frac{\ell}{A(E_m)}\right)(e_p) \qquad \text{Eq.5}$$

where:
A = cross-sectional area of the tape (0.0064 in^2 (assumed))
E_m = modulus of elasticity for steel (29,000,000 lb/in^2 (assumed))
ℓ = nominal tape length
e_p = error in tension (±5 lb)

For the tension error calculations:
ℓ = 200 ft
ℓ_s = 179.86 ft (last segment)

$$E_{e_t} = \pm\sqrt{\left\{\left(\frac{\ell}{A(E_m)}\right)(e_p)\right\}^2(3) + \left\{\left(\frac{\ell_s}{A(E_m)}\right)(e_p)\right\}^2}$$

$$= \pm\sqrt{\left\{\left(\frac{200.00}{(0.0064)(29\times10^6)}\right)(\pm5)\right\}^2(3) + \left\{\left(\frac{179.86}{(0.0064)(29\times10^6)}\right)(\pm5)\right\}^2}$$

$$= \pm\sqrt{0.000087089 + 0.000023478}$$

$$= \pm0.011 \ ft$$

Sag: Using the sag correction equation, Eq.6, and propagating the error for sag in each measurement segment by the law of compensation, the tension error related to sag, for the total length, is ±0.251 ft. (Using the derivative with respect to tension.)

$$E_s = \left(\frac{W^2\ell}{12P^3}\right)(e_p) \qquad \text{Eq.6}$$

where:
W = weight of the tape between supports
ℓ = length of the tape between supports
P = tension on the tape
e_p = error in tension

$$E_{s_t} = \pm\sqrt{\left\{\left(\frac{W^2\ell_1}{12P^3}\right)(e_p)\right\}^2(3) + \left\{\left(\frac{W^2\ell_s}{12P^3}\right)(e_p)\right\}^2}$$

$$= \pm\sqrt{\left\{\left(\frac{(5^2)(200.00)}{(12)(25^3)}\right)(\pm5)\right\}^2(3) + \left\{\left(\frac{(4.5^2)(179.86)}{(12)(25^3)}\right)(\pm5)\right\}^2}$$

$$= \pm\sqrt{0.053333 + 0.009433}$$

$$= \pm0.251 \ ft$$

- Temperature error - Using Eq.7 the total temperature error is ±0.050 ft. Remember, this error accumulated directly with the total measured length.

$$E_t = (K)(L)(e_t) \tag{Eq.7}$$

where:
K = coefficient of thermal expansion
L = total measured length
e_t = error in temperature reading (±10°)

$$E_t = (0.00000645)(779.86)(\pm 10)$$

$$= \pm 0.050 \ ft$$

- Plumbing and reading errors - These errors occur twice for each taping segment and accumulate according to the law of compensation. Therefore, the frequency, n, is 4 times. The total propagated random error is ±0.028 ft.

$$E_{pr} = \pm s_x \sqrt{2n}$$

$$= \pm 0.01\sqrt{(2)(4)} \tag{Eq.8}$$

$$= \pm 0.028 \ ft$$

- Alignment errors - These are not random errors and do not compensate; they accumulate and always make the measurement longer. Do not include these in random error calculations. Perform field procedures to eliminate this systematic error.

$$E_a = \frac{d_a}{2\ell} \tag{Eq.9}$$

where:
E_a = alignment error on tape length
d_a = lateral displacement
ℓ = length of tape

$$E_{a_t} = -\left(\frac{0.5}{2(200)}\right)(3) - \left(\frac{0.5}{2(179.86)}\right)$$

$$= -0.004 - 0.0014$$

$$= -0.005\ ft$$

- Marking the chaining points - This task occurs at the end of each full tape length; the fractional taping segment to monument B does not require the setting of a point. Therefore the frequency, *n*, is 3 times. The total marking error is ±0.017 ft.

$$E_m = \pm e_m \sqrt{n}$$

Eq.10

$$= \pm 0.01\sqrt{3}$$

$$= \pm 0.017\ ft$$

- Vertical angle reading errors - Each vertical angle reading was stated as having an error of ±2′. Using the error propagation formula from *Surveying Measurements and their Analysis*, the four segments produce errors of ±0.004, ±0.008, ±0.007, and ±0.025 ft, respectively. Then, using the standard formula for propagation of random errors, Eq.3, the total vertical angle error is ±0.027 ft.

Eq.11

$$E_s = \pm(\ell)(\sin \triangle_v)(\sin\ e_v)$$

$$E_{s_1} = \pm(200.00)(\sin\ 1°56')(\sin\ 2') = \pm 0.004$$

$$E_{s_2} = \pm(200.00)(\sin\ 4°09')(\sin\ 2') = \pm 0.008$$

$$E_{s_3} = \pm(200.00)(\sin\ 3°22')(\sin\ 2') = \pm 0.007$$

$$E_{s_4} = \pm(179.86)(\sin\ 13°41')(\sin\ 2') = \pm 0.025$$

Using Eq.3 the result is:

$$E_v = \pm\sqrt{0.004^2 + 0.008^2 + 0.007^2 + 0.025^2}$$

$$= \pm 0.027 \ ft$$

Once you have determined the independent effect of each measurement task or procedure, you can calculate the total error uncertainty on the measured line between monuments A and B. The total error is calculated using the individual errors previously determined by equations (5), (6), (7), (8), (10), and (11).

Therefore, using the general propagation formula, Eq.3, the total error uncertainty is:

$$E_T = \pm\sqrt{0.011^2 + 0.251^2 + 0.050^2 + 0.028^2 + 0.017^2 + 0.027^2}$$

$$= \pm 0.26 \ ft$$

The best estimate of the distance, with its uncertainty, would be 773.85±0.26 ft, or possibly, 773.8±0.3 ft depending on the confidence you have in the precision of your procedures, observations and standard errors.

The relative accuracy of this error uncertainty between points A and B is 1:2,976. If the client or project specifications required a higher positional accuracy, e.g., 1:5,000, then procedures would have to be modified so this higher accuracy could be achieved. With respect to the previous example, the following steps would be appropriate:

- Compute the desired level of error uncertainty from the specified accuracy. The total allowed error for 1:5,000 is 0.15.

- By applying the error propagation theory in reverse, compute the desired procedure change to produce an uncertainty error of 0.15 ft. The largest error in this taping problem is associated with the sag of the chain as affected by the 5 lb tension error. Working the total error uncertainty propagation formula (see above) with the desired resultant of 0.15 ft and having the sag error as the unknown will produce a desired sag error of ±0.135 ft.

- Then, using the formula to compute the total error for sag, Eq.6, with 0.135 ft as the desired result and the error in tension as the unknown, the error in tension computes to ±3 lb. Adding a tension handle to the taping procedure will provide the desired result.

- Increasing the repetitions of plumbing and reading, marking the points, or reading the vertical angles will yield about the same uncertainty because they are constrained by the systematic effect of the procedures and will produce little gain in the resultant error.

EDM measurements - WAC 332-130-100 (2) requires that all distance measuring instruments be compared and adjusted annually, at a minimum, to a National Geodetic Survey calibrated baseline.

The following primary sources of EDM errors are systematic:

- Manufacturer's instrument centering error
- Vertical tilt axis
- Prism constant
- Uncorrected atmospheric corrections
- EDM/theodolite/prism height relation
- Uncorrected refraction and curvature of Earth (long distances)

The following primary sources of EDM errors are random:

- EDM instrument centering error
- Prism centering error
- Manufacturer's accuracy specifications; (e.g., ±(5 mm + 5 ppm))
- Reading of atmospheric conditions
- Reading the vertical angle

These random errors must be computed according to the rules of error propagation to determine their effect on surveying measurements.

The random errors that will have the largest effect on the level of uncertainty of EDM measurements are EDM and prism centering errors and the manufacturer's accuracy specifications. Uncertainties from misreadings of atmospheric conditions would normally be negligible if properly corrected at each measurement. Refer to the vertical angle discussion on taped measurements, Eq.11, for the general effect on EDM measurements. Remember; the longer the measured distance and the steeper the vertical angle, the less certain is the horizontal measurement if care is not taken in the reading of the vertical angle.

If an EDM manufacturer's accuracy specification of ±5 mm is a constant for any measurement, then EDM measurements over short distances may not be precise enough to obtain a desired accuracy. For example, if the desired accuracy was 1:10,000, then 0.016 ft (5 mm) divided by the shortest distance possible to meet the desired accuracy should equal 0.0001 (1:10,000). The proportional error in parts per million is negligible for this example. The shortest distance computes to 160 ft. A distance of 50 ft measured with this EDM would have a relative accuracy of 1:3,125.

Example Problem 3.1.2:

A measurement is needed from point A to lay out a horizontal distance of 4000.00 ft to establish a required point B on an airport runway extension for flight clearance purposes. The ground is level for all practical purposes, so there is no vertical angle, and the distance to point B can be measured directly. The EDM manufacturer's specification is ±(5 mm + 5 ppm). The instrument and prism centering errors are ±0.01 ft and ±0.02 ft, respectively. The prism offset has been corrected for and is zero. Atmospheric readings have been entered in the EDM correctly and are negligible for this exercise.

What is the uncertainty of the position of point B with respect to point A?

The uncertainty of the measurement will be affected by the EDM manufacturer's specifications, the instrument centering error, and the prism centering error. Each one will occur once and will accumulate by the general rule of propagation of random errors. The total error uncertainty as computed using the general propagation formula, Eq.3, is:

$$\begin{aligned} E_T = E_{s_x} &= \pm\sqrt{0.036^2 + 0.01^2 + 0.02^2} \\ &= \pm 0.04 \ ft \end{aligned}$$

The reported uncertainty of the distance to point B with respect to point A is 4,000.00±0.04 ft. Therefore, the positional uncertainty of point B with respect to point A is ±0.04 ft. The relative accuracy between the two points will be 1:100,000.

3.2 Errors in Direction

The following discussion focuses on instrument and procedure errors. Direction errors can also be caused by other factors such as, astronomical observations, positional errors on existing backsights and foresights that are used to determine beginning and closing azimuths, Earth curvature, and meridian convergence.

The measurement of a horizontal angle with a theodolite is influenced by many sources of error, some instrumental, some personal, and some from natural causes. Detailed discussions are presented in Chapter 6 of *Surveying Measurements and their Analysis* and Section 7.2 of Mikhail and Gracie's (1981) *Analysis and Adjustment of Survey Measurements*.

Most instrument errors are systematic. They can be compensated for by proper procedures, maintaining the instrument in proper adjustment, and performing an equal number of direct and inverted observations.

The primary sources of instrument errors are misalignment of the:

- Collimation axis
- Vertical axis
- Horizontal axis
- Optical plummet axis

Most personal errors are characteristically random. If standard procedures are developed and adhered to, then standard deviations can be used for each task throughout the survey. The primary sources of personal errors are inaccurate:

- Focusing
- Reading
- Pointing
- Centering the theodolite
- Centering the target
- Centering the bubble

Most natural errors are caused by the weather, and they influence most personal errors. The effects of these errors are difficult to quantify. The best way to deal with natural errors is to be constantly aware of them so as to minimize their effects. Some examples are:

- Pointing errors caused by wind, haze, heat waves, or refraction (when the line of sight passes too close to objects),
- Centering and leveling errors caused by the thawing of frozen ground,
- Reading errors caused by poor lighting.

To determine the final random error associated with the angle measurement, close attention must be given to each measurement task. Some tasks, such as reading and pointing, will each give some expected resultant error and, in general, are not affected by the length of the measured line. In contrast, standard centering errors for the theodolite and targets will result in varying sizes of errors in the resultant angle, depending on the length of the line measured and size of the interior angle.

Example Problem 3.2.1:

Two interior angles of approximately 180° each, were turned with the same precision, equipment, and procedures. One angle had backsight and foresight distances of 200 ft, and the other angle had distances of 1,000 ft. The following derived standard errors in this example are the same for each angle. Not all random errors are considered.

Given (experimentally derived values):

Reading: ±3.5″
Pointing: ±2.0″
Target centering: ±0.02 ft
Instrument centering: ±0.015 ft

What is the expected angular error at each set-up?

- Reading (2 angle sets with directional theodolite):

  Each set consists of two angle measurements, one with the telescope direct and one with it inverted. The error in reading is equal to the standard error in reading, ±3.5″, times the square root of the number of sets, 2, divided by the square root of the number of angle measurements, 4. See Eq.4 in Section 2.6 for the basic formula.

$$E_r = \pm\frac{(3.5)(\sqrt{2})}{\sqrt{4}} = \pm 2.5''$$

(This result is the same for both set-ups.)

- Pointing:

  Because both the reading and pointing errors occur with each pointing, you can use the same formula and values as for the reading computations. The error in pointing is equal to the standard error in pointing, ±2.0″, times the square root of the number of sets, 2, divided by the square root of the number of angle measurements, 4.

$$E_p = \pm\frac{(2.0)(\sqrt{2})}{\sqrt{4}} = \pm 1.4''$$

(This result is the same for both set-ups.)

- Target centering:

  The maximum angular error would occur when the target centering error is perpendicular to the direction of each pointing of the theodolite. Each target centering error for the backsight and foresight will affect the angular error and will have to be propagated by the law of addition of random errors. Once the target has been set over the point, no number of repetitions will change the effect of the target centering error. The target centering errors are random in nature, but they are systematic in effect, unless the targets are recentered with each pointing.

Because the backsight and foresight distances are equal, the angular error in target centering of the backsight will equal the angular error in target centering of the foresight. This would not be true if the backsight and foresight distances were not equal. The maximum angular error in target centering for the mean horizontal angle is equal to the square root of the sum of the squares of the angular errors from the backsight and foresight targets. The value of 206265″/radian is used to convert the answer from radians to seconds.

For sights of 200 ft:

$$E_{t_1} = E_{t_2} = \frac{(0.02)(206265)}{200} = 20.6''$$

$$E_t = (20.6)(\sqrt{2}) = \pm 29.1''$$

For sights of 1,000 ft:

$$E_{t_1} = E_{t_2} = \frac{(0.02)(206265)}{1000} = \pm 4.1''$$

$$E_t = (4.1)(\sqrt{2}) = \pm 5.8''$$

- Instrument centering:

Instrument centering errors are similar in nature to target centering errors: systematic in effect, but random in nature. Once the instrument has been set over the point, no number of repetitions will change the effect of the instrument centering error. Refer to Figure 6.11 on page 177 of *Surveying Measurements and their Analysis* for a good depiction of the effect of theodolite centering errors on angles.

Because the backsight and foresight distances are equal, the following formula will be used. See page 178 of *Surveying Measurements and their Analysis* for the formula and the definition of terms:

For sights of 200 ft:

$$E_i = \pm\left(\frac{(0.015)(\sqrt{2})\left(\sin\frac{180}{2}\right)}{200}\right)(206265) = \pm 21.9''$$

For sights of 1,000 ft:

$$E_i = \pm\left(\frac{(0.015)(\sqrt{2})\left(\sin\frac{180}{2}\right)}{1000}\right)(206265) = \pm 4.4''$$

- Combined set-up error (uncertainty of angle):

  The combined angular error follows the law of addition of random errors. See Section 2.6, Eq.3, for the basic formula.

  For sights of 200 ft:

$$E_T = \sqrt{2.5^2 + 1.4^2 + 29.1^2 + 21.9^2} = \pm 36.5''$$

  For sights of 1,000 ft:

$$E_T = \sqrt{2.5^2 + 1.4^2 + 5.8^2 + 4.4^2} = \pm 7.8''$$

Comparing the errors for the different sight distances shows the need for care when centering targets and instruments on short sight distances. If standard procedures were followed throughout a survey and lengths of traverse legs were varied as in the above example, then weighted adjustments to the angles based on the individual uncertainties would be appropriate and justifiable. After analysis, as a means of increasing final accuracy, you may consider modifying procedures to reduce uncertainties for short sight distances.

A combined angular error of ±36.5″ or ±7.8″ represents one standard deviation; that is, 68.3% of the time the results of the survey and analysis will fall within the calculated ranges. Typically, the surveying profession demands that the level of certainty be 90% or 95% instead of 68.3%. To obtain the corresponding range of acceptable values, the standard error (one standard deviation) should be multiplied by 1.64 for a 90% confidence level or 1.96 for a 95% level of certainty. In this example, multiply ±36.5″ and ±7.8″, respectively, by 1.96 to obtain ±71.5″ and ±15.3″ for the corresponding acceptable range of values for sights of 200 and 1,000 ft with 95% confidence. If these values are unacceptably large, the surveyor should use the methods for controlling error magnitudes as described in Section 9-4, *Surveying Measurements and their Analysis*.

3.3 Errors in Position

An error in position is the combined effect of the uncertainty in distance and in direction. This uncertainty will continue to increase through a traverse as the errors accumulate in accordance with random error theory. Error in position is not the same as error of closure. An error of closure may fall anywhere from zero to the full amount of the expected uncertainty of the final position. Only the accepted theories on probability and errors can help estimate the uncertainties in expressed positions (coordinates).

The previous discussions dealt with random errors of measurement in one dimension only. A random error in distance measurement will cause an uncertainty in position along the direction of the measurement, creating one axis of an error ellipse. A random error in angle measurement will cause an uncertainty in position perpendicular to the direction of the measured line, creating the other axis of the error ellipse. The combination of these two random errors creates the shape and direction of the error ellipse. In a normal traverse, rarely will the axes of the ellipse be of the same magnitude. The larger error will be the major axis of the ellipse and the smaller error will be the minor axis. The real world of boundary surveying controls the resultant precision of the measurements and, therefore, the size, shape and direction of the ellipse. These error ellipses will change in direction and magnitude as the random errors accumulate.

In most instances the station error ellipses will be related to a particular point in a survey as chosen by the surveyor. Figure 3.3.1A shows the error ellipse at B with respect to A; Figure 3.3.1B shows the error ellipse at C with respect to B. Figure 3.3.1C shows the combined error ellipse at point C with respect to point A as a result of combining the relative errors of courses AB and BC.

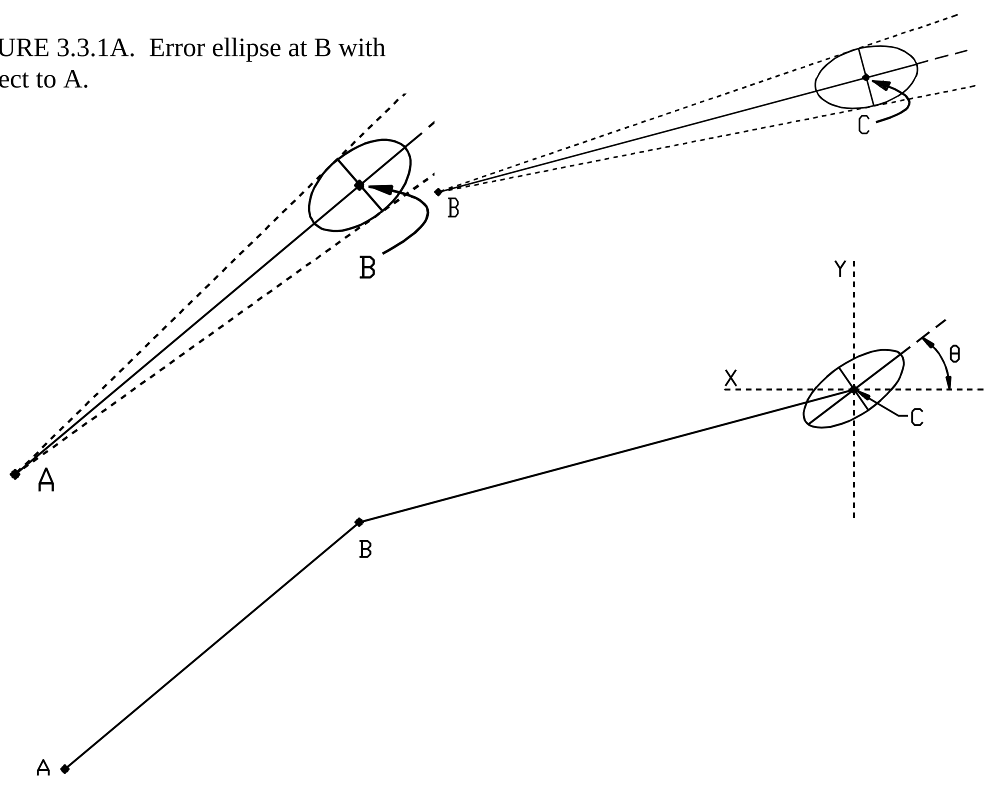

FIGURE 3.3.1A. Error ellipse at B with respect to A.

FIGURE 3.3.1B. Error ellipse at C with respect to B.

FIGURE 3.3.1C. Error ellipse at C with respect to A.

In general, progression through a traverse will combine errors; the axes of the error ellipses for the second and succeeding courses are not likely to be parallel with and perpendicular to the measured lines. The position of the major and minor axes of the error ellipses are located by a rotation angle θ with respect to the coordinate axes.

The region inside each error ellipse represents probable positions, statistically determined at predetermined levels of certainty, for each corresponding point along the traverse. For example, if the surveyor wishes to know the region within which the position of a point is likely to be found with 95% certainty, the corresponding error ellipse will be larger than one that reflects a certainty of only 50%. Table 2 provides multipliers showing the ratio of ellipse dimensions for various probabilities that the point being considered lies within the error ellipse. With reference to that table, the lengths of the major and minor axes of the 95% error ellipse will be 2.447/1.177 = 2.079 times those of the 50% error ellipse.

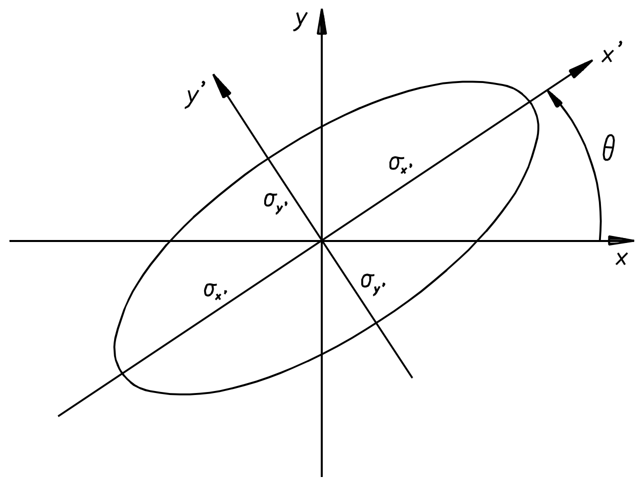

FIGURE 3.3.2. Typical error ellipse.

A typical error ellipse is shown in Figure 3.3.2. In general, the principal axes of the ellipse, x' and y', do not coincide with the survey coordinate axes, x and y. The direction of the error ellipse is represented by angle θ rotated from the survey axis x. Typically, a positional error of a traverse point is represented as $\sigma_{x'}$, $\sigma_{y'}$ and rotation of the axes, θ.

The error analysis of an individual point relative to some beginning point is just one way to examine your work. The further comparison of the relative errors between adjacent property corners is of more importance and will be the subject of the next chapter.

4. RELATIVE ACCURACY ANALYSIS

4. Relative Accuracy Analysis

Surveying is "The science and art of making such measurements as are necessary to determine the relative position of points above, on or beneath the surface of the earth, or to establish such points in a specified position." The science of surveying involves the application of mathematics and physics. The art of surveying involves the judgment and logical thinking that goes into pre-analysis and post-analysis. Pre-analysis includes the planning and laying out of a project and proper selection of equipment, methods, and procedures. Post-analysis includes analysis of data and making correct decisions and appropriate adjustments.

Science and art must be properly combined to derive the relative positions of points to other points or a specified position on the earth. The accuracy of a position's location or measurement is directly related to the surveyor's understanding of systematic and random errors, direct and indirect measurements, significant figures, the difference between precision and accuracy, the method and procedures employed, and the analysis of the data. Control is evident if the surveyor can, with confidence, state the probable numerical range of each measurement along with the level of certainty (percent of probability) related to this range.

If every measurement has a certain degree of uncertainty, then how does the surveyor recognize and control the uncertainty of those measurements? If a client needs a property line established so that a 50-story building can be built at zero setback, how can the surveyor design the survey measurement system that will assure the building is not built over the line? Many surveyors would arbitrarily select equipment and methods, then merely hope that the accuracy was achieved by relying on the least count and other elementary precision concepts. Furthermore, a surveyor might certify to the accuracy in the absence of any quantitative support. Pre-analysis can help in determining the equipment and traverse configuration that will attain the desired accuracy. Post-analysis provides the basis for quantitative evaluation.

4.1 Selection of Survey Equipment

Surveyor's today have a vast array of instrumentation available to measure angles and distances. They must know the limits of equipment selected for the job, and they are charged with selecting instrumentation and procedures that will attain the accuracy demanded for a particular project. The choice of such instrumentation and procedures must withstand statistical tests and the scrutiny of fellow professionals.

Instrumentation available to the surveyor today includes the following:

- Staff compass and cloth tape
- Compass theodolite (Wild T-0) and steel tape
- Transit and steel tape (30″-transit, 1′-transit)
- Theodolite and EDM
- Theodolite/EDM total station

- Inertial positioning systems
- Global positioning systems (GPS)
- Photogrammetric systems
- Various combinations of the above

A prudent surveyor will not choose a compass and tape and probably not a 1′-transit and chain to locate a property line for the 50-story building. Appropriate pre-analysis of the relative accuracy capabilities of this equipment would show error ellipses larger than the accuracy required. Surveyor's should be aware of the differences in the relative accuracy of a point set using a 1′-transit, 30″-transit and various theodolites. They should also be aware of the precision range for a variety of angular readings, both direct and reversed.

Surveyor's should know the manufacturer's stated precision for the equipment used or should make tests to determine its capabilities and limitations (manufacturers' instrument specifications/precisions are regularly published in *P.O.B.* magazine). For example, a surveyor might use an EDM instrument to measure a line that is to be 300.00 ft. The EDM manufacturer's published standard deviation for that distance is ±0.02 ft. The surveyor can be 68.3% sure that the measurement is between 300.02 and 299.98 ft. There is also a 99% certainty that the measurement is between 300.05 and 299.95 ft.

Instruments must be maintained in close adjustment according to the manufacturer's specifications and shall be compared and adjusted at least annually to a calibrated base line (see WAC 332-130 and the Technical Memorandum, *Use of Calibration Baselines*, published by the National Oceanic and Atmospheric Administration in 1977).

4.2 Selection of Procedures for Using Surveying Equipment

The public expects a professional surveyor to provide appropriate accuracy at a reasonable cost. Few surveying projects are alike. Therefore, surveyors will not always use the same equipment and procedures. Analytical thinking and professional judgement are always involved. The number of repetitions for the selected equipment, level of care taken in making readings and handling of equipment, and manner of making independent checks affect the time required for the survey, which affects cost. The challenge to a surveyor is in choosing the methods that satisfy the accuracy requirements.

Surveyors must analyze anticipated survey procedures to ascertain that they meet the precision requirements of the project. For surveys of a similar nature, a proven procedure may be used repetitively. In more complex surveys or where procedures have not been established, it is prudent to develop a plan for equipment use and analyze its effect throughout the survey. If the predicted precision is satisfactory, the procedure is acceptable; if too low, the procedure must be improved; if too high, certain steps of the plan may be moderated. Several attempts may be necessary before deriving a plan for equipment use that provides the desired result.

The procedures you chose should:

- Help avoid mistakes.
- Be designed to produce the desired accuracies.
- Spell out what to do in a given situation; however, specific procedural guidelines will seldom cover all situations.
- Be modified with the use of different equipment.
- Be developed by each professional, based on their own unique situation, for use by technicians.
- Include calibration of the instruments to be used.

You may need to vary procedures and methods due to the scope of a job, weather, terrain, equipment, and differing traits of survey personnel.

As stated in *Surveying Measurements and their Analysis*, making a measurement can be reduced to a five step procedure:

- Selecting the method or technique to use for a specific measuring task.
- Deriving and adopting specifications and standards.
- Executing the measurements and recording the data.
- Employing various checks and controls during execution.
- Analyzing and reducing the data.

4.3 Configuration of the Survey

Authors of many texts and surveying articles have stated that an expression of lineal error of closure is invalid as an indicator of accuracy and is only a statement about precision. The American Congress on Surveying and Mapping (ACSM), in its publication *Classification and Specifications for Cadastral Surveys*, states that its procedures assure only precision. The configuration of the project scheme, number of traverse legs, steepness of the terrain, line lengths, measurement intervals and closure points all affect the accuracy of the survey.

Many factors, such as the weather, method of doing backsights and foresights, ability to measure accurately on steep slopes, and instrument reading, pointing, and set-up errors influence the expected traverse closures. These sources of errors and many others tend to accumulate as the opportunity to occur increases. The configuration of the traverses as determined by the direction, length, and number of traverse legs in each survey project, when combined with the random errors determined from the procedures, will result in a wide variety of positions (error ellipses; see Section 3.3) for the set corners. It is not necessary to go through an actual mathematical analysis to know this will happen. But it is necessary to do the analysis to determine the magnitude of the probable errors.

Once the magnitude of probable errors is known, appropriate questions can be asked: What will the positional relationship be between different surveys? How far apart will two surveys be?

Are the public's and the profession's interests best served by multiple monumentation of a corner? Or maybe the more pertinent question is, "Is one survey any more correct than the other?"

Computer software is available to assist the surveyor in answering these questions. We now consider an actual survey in which a commercial software package is used to identify relative accuracies of the contained points.

Example Problem 4.3.1:

An analysis of an actual job will show the magnitude of probable errors and how they may affect the position of set corners. Figure 4.3.1 shows a section subdivision survey in a rural forest location that was done with a 10″ total station with an EDM specification of ±(5 mm + 3 ppm). Standard defaults were used (10″ for the angles and 0.005 ft for tribrach centering errors). The least squares program of STAR*NET, version 3.06, from Starplus Software, Inc. was used for the analysis. The results are reported at the 95% confidence level. Three individual traverses were looped and connected together and have unadjusted traverse closures of 1:10,280, 1:37,207, and 1:49,349.

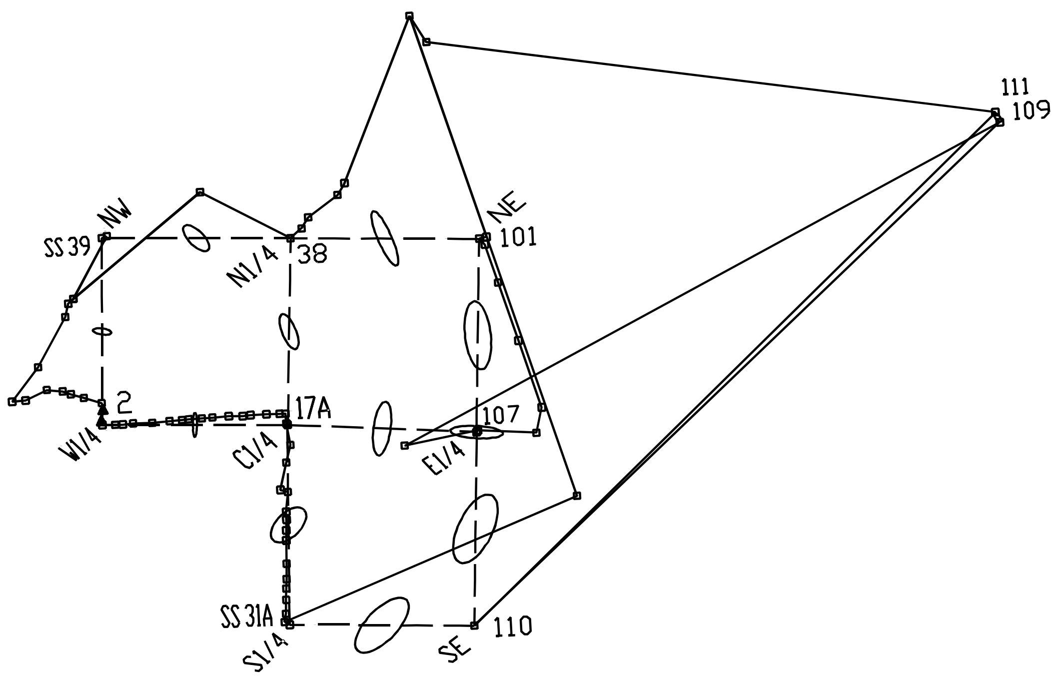

FIGURE 4.3.1. Traverse layout with error ellipses. (Not to scale)

The station coordinate error ellipses in Table 4.3.1 have been selected from the complete computer printout to provide a feel for the accuracy of the survey. The error ellipses are relative to the traverse point of beginning near the west 1/4 corner.

TABLE 4.3.1

Station	Pt #.	Major Axis	Minor Axis	Azimuth of Major Axis
W 1/4 cor	2	0.022 ft	0.021 ft	95°03′
NW cor	SS39	0.987 ft	0.283 ft	101°08′
N 1/4 cor	38	1.623 ft	0.285 ft	147°13′
E 1/4 cor	107	2.577 ft	0.824 ft	12°58′
NE'ly point	109	6.450 ft	0.550 ft	168°00′
SE cor	110	3.127 ft	1.308 ft	40°49′
S 1/4 cor	SS31A	2.165 ft	0.739 ft	61°03′
C 1/4 cor	17A	1.294 ft	0.114 ft	3°46′

This program also provides relative error ellipses between preselected points on the project. This allows the surveyor to analyze the relative accuracy (precision) between found and set corners throughout the survey. The software provides this information when key corners are selected (See Table 4.3.2).

TABLE 4.3.2

Stations			
From	To	Distance	Precision
W 1/4 cor	C 1/4 cor	2597.63 ft	1:2007
N 1/4 cor	C 1/4 cor	2578.37 ft	1:1303
S 1/4 cor	C 1/4 cor	2761.84 ft	1:1194
E 1/4 cor	C 1/4 cor	2658.95 ft	1:923
W 1/4 cor	NW cor	2580.64 ft	1:2613
NW cor	N 1/4 cor	2649.84 ft	1:1476
N 1/4 cor	NE cor	2647.86 ft	1:843
NE cor	E 1/4 cor	2673.49 ft	1:923
E 1/4 cor	SE cor	2672.38 ft	1:676
SE cor	S 1/4 cor	2589.04 ft	1:688

Version 3.06 of the STAR*NET program analyzes in the worst case scenario which is based upon distance only and disregards the direction of the semi-major axis. While the new versions will analyze in a way that will provide more realistic relative precision values, the above information is still quite useful. It still tells us that linear error closures have no bearing on the ability to place corners accurately and that a survey has varying degrees of relative accuracy throughout the project.

The configuration of the survey plays a more important role in determining the position of corners than do good linear error closures. Long indirect traverses combined with long radial ties to corners will have a major effect on the surveyor's ability to accurately position corners. See, for example, points 109, 110, and 111 in Figure 4.3.1. An analytical least squares program can provide the data necessary to show the real results of the survey, the relative position of the corners. For a complex traverse survey, such a program may be the only way to obtain these results.

Example Problem 4.3.2:

This example is taken from a boundary survey control traverse performed in a typical rectangular city block with a monument at each of four centerline intersections (Figure 4.3.2). It was not possible to sight down two of the centerlines due to left-hand turn pocket lanes and truck traffic, so traverse points were set over on the sidewalks.

Points 61, 62, 65, and 66 are found monuments. A total station (5″) was used for angles and distances ±(3 mm + (3 ppm x D)). The raw data were quickly calculated in the field to check for closure; the closing error ratio was a surprisingly high 1:296,000. Knowing the difficulties with traffic and other influences, we suspected that the high closing error ratio was a fluke; we expected a traverse analysis would tell a different story. One might be tempted, after computing the closing error ratio, to think that any point on the traverse would be within 0.01 ft of the calculated coordinate.

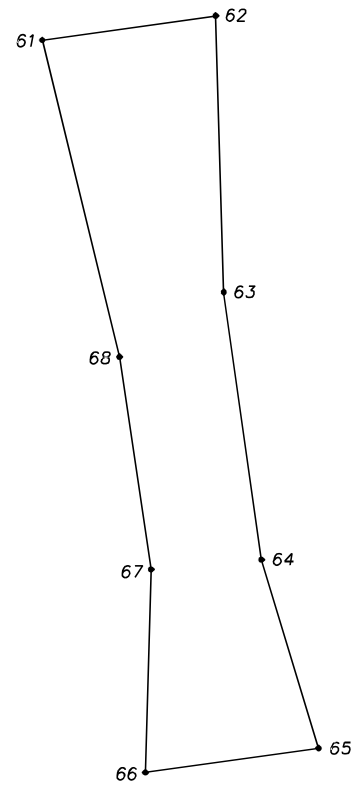

FIGURE 4.3.2. Original traverse of city block.

An analysis was made using <u>only</u> the instrument manufacturer's specified standard deviations for angle and distance measurements. The coordinate for point 62 was fixed (zero error). The resulting error ellipse for point 66 is shown in Figure No. 4.3.3.

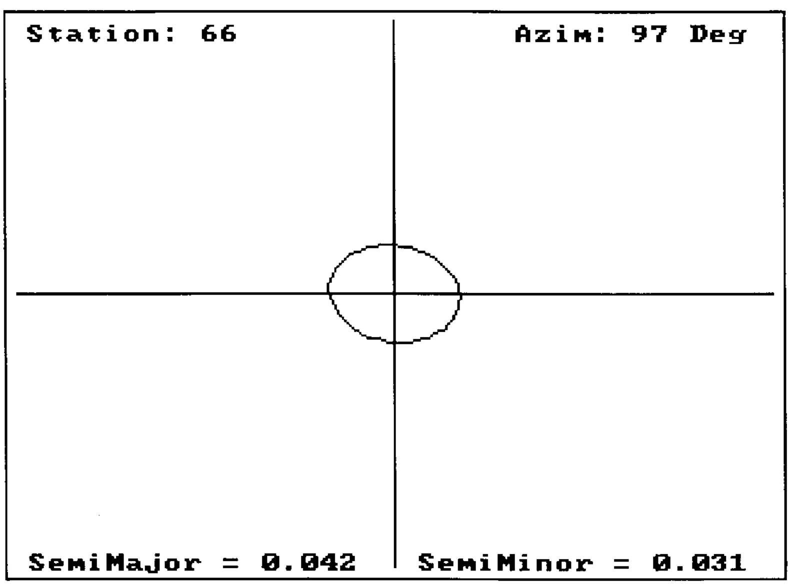

FIGURE 4.3.3. Error analysis considering only manufacturer's specifications.

Notice that at the far end of the traverse, at point 66, there is an error ellipse with major and minor axes measuring 0.04 ft by 0.03 ft respectively (relating to zero at point 62), quite an enlargement over the 0.01 ft closing line measurement.

There are other unaccounted for sources of random error in this traverse. These errors, when accounted for, will have the effect of enlarging the error ellipses even more and must be part of the overall analysis. When considering the random error expected from centering the instrument and centering the backsights/foresights, calling on our experience with and knowledge of the traverse methods and equipment used, we suspected that both centerings were probably no closer than 0.015 ft. We ran an analysis accounting for the instrument errors (above) and centering errors of 0.015 ft. The results are shown in Figure 4.3.4; the larger error ellipse around point 66 is now 0.21 ft by 0.08 ft. We elected to reevaluate potential instrument error and returned to the field to retraverse the same control points.

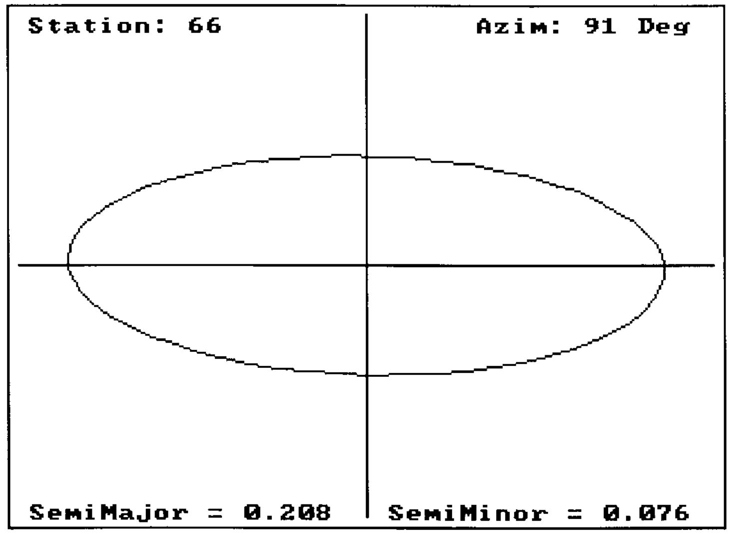

FIGURE 4.3.4. Error analysis considering manufacturer's specifications and centering errors.

First, the total station EDM was checked on an NGS baseline (see *Use of Calibration Baselines)*. The optical plumbs on the tribrachs were carefully adjusted insuring better instrument centering as well as better centering for the backsights and foresights. We estimated that we could now center the instruments and backsights/foresights to within 0.01 ft. We retraversed the points at a time of day when traffic and heatwaves were less of an influence.

Again, the raw data for the second traverse (see Figure 4.3.5) were calculated in the field to check for major blunders. We found a closing line length of 0.02 ft and a closing error ratio of 1:102,000. Comparing closing error ratios, the quality of the second traverse seemed to be less than that of the first, even though we had taken greater care with the second. But an analysis of the second traverse revealed that the second traverse was indeed much better than the first.

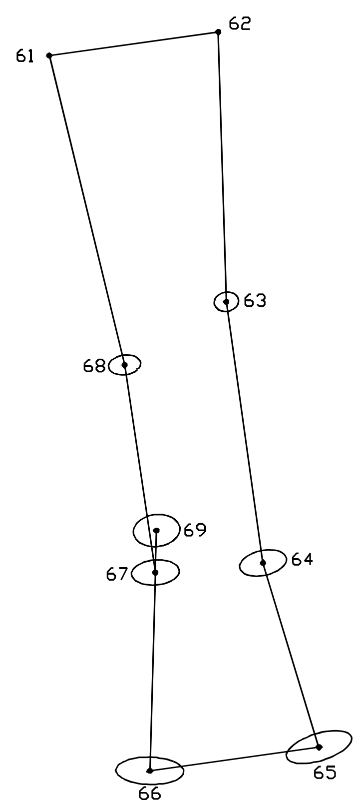

FIGURE 4.3.5. Retraverse of city block with point ellipses.

Figure 4.3.6 shows that the error ellipse around point 66 is substantially smaller than shown on Figure 4.3.4 indicating that the quality of the second traverse is statistically better than the first traverse. We are *uncertain* where in the region described by the boundaries of the ellipse the coordinate falls, but we are 95% *certain* it falls somewhere within those boundaries. This example shows that closing error ratios are no indication of traverse quality--they may only be used to indicate major blunders.

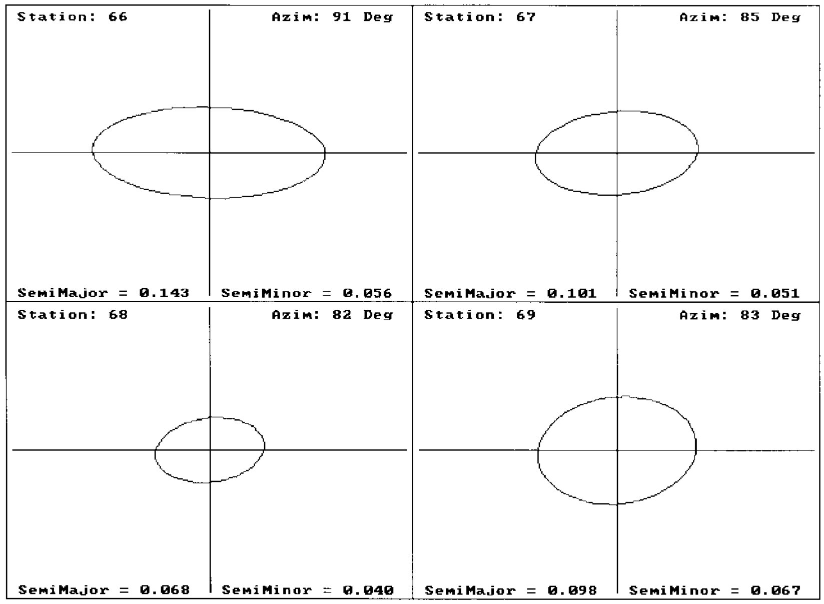

FIGURE 4.3.6. Error ellipses of retraverse of city block.

The second traverse was used to set point 69. Note the ellipse around point 69 on Figure 4.3.6. We are 95% certain that the point was set within the region described by the 0.10 ft by 0.07 ft error ellipse.

Figure 4.3.7 shows an error ellipse on the line between points 65 and 66, as well as on other lines. The error ellipses shown in Figure 4.3.5 were around points, relating each point to point 62, which we chose to fix with no error. The relative error ellipses in Table 4.3.3 and in Figure 4.3.7 show the relative accuracy of point 65 with respect to point 66 and the other point pairs. By knowing the relative accuracy of any two points in a traverse, it will be easier to predict and control the quality of work performed from those points in the future.

TABLE 4.3.3

Stations		Axes		Azimuth of
From	To	Major	Minor	Major Axis
62	63	0.051 ft	0.040 ft	79°
63	64	0.064 ft	0.040 ft	75°
64	65	0.061 ft	0.039 ft	63°
65	66	0.056 ft	0.042 ft	172°

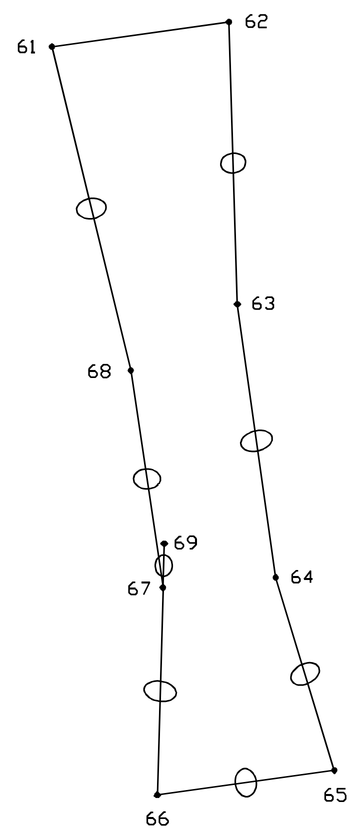

FIGURE 4.3.7.
Retraverse of city block with relative ellipses.

In review, the traverse analysis included more than just linear error of closure. We have shown that considering and including all errors is necessary in an analysis. The analysis should provide realistic knowledge of the probable errors at each point. If only standard manufacturer's specifications are considered, the associated error ellipses will be unrealistically small. Error ellipses can be made acceptable only by selecting, adjusting, and calibrating equipment and adopting better procedures. Any monument set from a traverse will fall within the region of the associated error ellipse.

Before we learned about relative accuracy and the type of traverse analysis discussed in this guidebook, we readily accepted very high closing error ratios without question. Yet, if we continued to use the traverse and extend it, we would experience difficulties. Today we know why.

Once you have informed your field and office personnel about your new system of quality control, which includes analysis software, the time spent processing raw data for an analysis and for adjustment is no more than for any other data entry system. You will soon find that an analysis program is a tool you can rely on for the short simple traverses as well as the extensive traverse network.

4.4 Adjustment of Surveys

Before any adjustment begins, you need to detect and remove all systematic errors and mistakes. Only random errors should remain. The actual closure normally should not exceed the expected closure. If it is greater, you may need to examine the survey measurements and network design to locate any problems. The expected closure is determined by pre-analysis of the survey system as computed by random error theory. The expected closure error depends on the accumulation of errors in individual distance and angle measurements, as well as general traverse configuration, number of set-ups in the traverse, and the lengths of the traverse lines.

Adjustments are not part of an analysis. They simply make the values close mathematically. The final adjusted values are no freer from error than the original field data, but they do provide a more probable value (coordinates) for the position of the points. The most important thing to remember is that the adjustment should not shift the position of any point more than the theoretical uncertainty.

The adjustment method should be chosen with reason and logic to suit the surveying conditions. Generally, a simultaneous least-squares adjustment for direction and distance will be satisfactory, though it is not justified in many cases. If the uncertainty has been caused by direction and distance in equal proportions and is a function of the distance, the compass rule will be satisfactory. Multiple interconnecting traverses (crossed lines and junction points) also add complexity to the calculations and are normally approached one loop at a time until the last traverse is adjusted. This "last traverse adjustment" always brings anxiety to the error closures and usually causes reductions in reported precision. In reality the last traverse's precision is probably similar to the other traverses.

The compass rule could be an appropriate adjustment method for a single loop traverse closing on itself or a traverse between two known points, when the directions and distances have been measured with comparable precision. However, the compass rule method is not adequate for analyzing a survey. Least-squares adjustment is helpful for surveys that have redundancy, contain varying precision or field conditions (weight factors), have multiple interconnecting traverses, or require greater precision. An analytical least-squares adjustment program can give you an itemized breakdown of your survey, showing exactly how each of your field observations fits into the overall survey. The adjustment will be a simultaneous best-fit solution and will eliminate the "last traverse adjustment" problem.

Several tools have been provided for your use in Relative Accuracy Analysis. Equipment selection, procedures and traverse configuration all play key roles in the end product and the adjustment process utilized. How you use this analysis in your survey practice has not been covered by the proceeding chapters. The practical daily analysis of a property corner's relative position is the focus of upcoming chapters. You have been provided with a foundation from which to move forward.

5. USE OF RELATIVE ACCURACY

5. Use of Relative Accuracy in Surveying Practice (This chapter will be completed in future supplements.)

You may be thinking that the analysis procedures are laborious and will add substantially to the cost of your surveys. There is no question that they add time to the process and that suitable software is necessary. The bulk of the input data, however, is easily and quickly recorded by the field personnel. After the procedures are in place, the impact on your operation time will be minimal, and your results will be both predictable and sound.

5.1 Acceptance or Rejection of Existing Corners

With these analysis procedures surveyors have an additional and powerful tool to use when accepting or rejecting existing monuments. Although a monument is a physical object intended to mark a corner location, it may not represent the true position. Accepting existing monuments depends upon the verification of their authenticity or their falling within the acceptable range of relative accuracy. Verified original corners should not be analyzed for acceptability through relative accuracy procedures. The basic legal principles of surveying are not affected by these procedures. Depending on the equipment and procedures used, an analysis of the measurements should reveal the mathematical reliability behind a decision to accept or reject the found point.

Surveyors daily compute coordinates of points from record data in order to traverse to them. The traverse coordinates versus the record data coordinates are useful in determining whether to accept the found evidence or reject it and set another point. But how reliable are your traverse coordinates? For example, if you find a nonoriginal monument 0.9 ft from the computed position, you may be tempted to reject it. If you compute the relative position error of your traverse at that location, you may find the monument is within the perimeter of your error ellipse. You may be left with two choices: accept the monument, or review your traverse analytical data to find possible areas of improvement. A smaller ellipse may make your decision making a bit easier. It would be helpful to know that the previous surveyor used standard prudent care in establishing the monument. If both surveys complied with the standard practices of the times, the monument probably should be accepted.

The need for professional judgement is not removed by having followed the procedures described in this guide. These analysis procedures are tools to be added to those you are using now. These tools, however, furnish you with a sound foundation for decision making. The relative accuracy analysis is probably more admissible as evidence in court than your instinct. Accepting or rejecting a monument on the basis of this type of statistical analysis is a masterful approach for the Professional Land Surveyor.

5.2 Future Considerations

The remainder of this chapter will be completed as a supplement for future distribution. The following proposed topics and questions will be addressed by the Relative Accuracy Committee to provide guidance in the use of relative accuracy in actual surveying practice. After reviewing this initial publication, please send questions and ideas that you believe should be addressed to any member of the Relative Accuracy Committee or DNR staff (see list on the inside of the front cover).

Determining encroachment through relative accuracy - If your survey shows an encroachment of 0.1 ft on a line controlled by corners that have an error ellipse of 0.2 ft, do you have an encroachment?

What to do with old and recent work - How far does a surveyor have to go to retrace old work? Does it have to be recomputed? If old work is retraced and found to be out of limits with the standards of that time, should it be rejected without question? If recent work is found to be out of limits with current standards, should it be rejected? What is the standard of rejection?

Reporting of relative accuracy - What would you want to know to adequately evaluate another surveyor's work? "If good measurement accuracy is achieved, part of its value is lost unless the estimated uncertainties are expressed in some way" (See Section 9-6, *Surveying Measurements and their Analysis*). What type of statement or data should appear on the map in relation to the uncertainty of the bearings and distances or corner positions? How will reporting of the uncertainty of measurements affect the writing of land descriptions?

How will relative accuracy be applied to a calculated corner? - What is the degree of uncertainty of a calculated corner? How does one compute this uncertainty?

Procedural changes resulting from relative accuracy - What changes, if any, need to be made to field and office procedures regarding implementation of relative accuracy?

Can there be an unacceptable error ellipse on a found and tied original monument? If so, does this unacceptable error ellipse have anything to do with the decision to accept the monument position?

Should positional tolerance ever replace closure as the standard for Washington State surveys?

APPENDIX

Selected References

The following books and publications are a sample of those available that address the subject of error analysis. Most survey texts published within the last 40 years will provide similar discussions. This list is not intended to be inclusive or an endorsement.

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see *P.O.B.*, *Professional Surveyor* and other publications for periodic equipment and software surveys.

Glossary

Accuracy. The agreement of a measurement or (the average of a set of) measurements with the true value.

Blunders. See mistakes.

Confidence interval. The range of measurement values that are acceptable at a given level of confidence or certainty.

Confidence level or level of certainty. A numerical value representing the acceptable certainty or desire on the part of the observer that a measurement is correct. For example, a confidence level of 95% reflects a desire that the observer be correct 95 times out of 100.

Degrees of freedom. The difference between the number of times a quantity has been measured and the minimum number required to uniquely determine the numerical values in the underlying mathematical model. For example, a distance measured four times which requires one measurement to estimate its length produces three degrees of freedom.

Error. The difference between an observed or computed value of a quantity and the true value of that quantity.

> **Random errors** - unavoidable errors, typically small and as likely to be positive as negative.
>
> **Systematic errors** - mathematically determinable errors that conform to known physical laws.

Error ellipse. The enclosed region contains the most probable location of a surveyed point at the center of the ellipse and all the other acceptable locations for the point at a given level of certainty. The higher the required level of certainty that the true point be contained in an error ellipse, the larger the dimensions of the ellipse will be.

Law of compensation. The principle that random errors tend to cancel, but never do so completely; errors accumulate in proportion to the square root of the number of opportunities for their occurrence. This also applies to an error in a series.

Mean, arithmetic. The sum of the values of individual measurements of a quantity divided by the number of measurement repetitions.

Mistakes. Blunders caused by the observer (false reading) or by a defective instrument, or both.

Population of data. All of the potential measurements of a particular quantity. The surveyor typically samples from the population of a data set when measuring a line or angle several times, determining only a small finite number of the infinite number of potential observations.

Positional accuracy. The comparison of the position of a surveyed point with the true location of that point. The statistical analysis of survey measurements shows the probable position of a point relative to a selected base point and line. See Relative Accuracy; the terms are often used interchangeably.

Positional error. The difference between the location of a point determined by surveying and the true position of the point. The error may never be known because in many instances the only way to determine the true position is by an infinite number of measurements. The true position of the point is then approximated as the mean position of a finite number of measurements, and error is correspondingly approximated by the difference between a surveyed position and the best estimate through all available measurements.

Positional tolerance. The precision required for the known, acceptable position. A tolerance standard that requires a 99% certainty of knowing the position of a point can require more precise measurements and greater expense than a 90% certainty of position.

Precision. The extent of agreement among measurements of the same quantity. An estimate of standard deviation is commonly used to indicate precision.

Propagation of random errors. A mathematical formulation by which accumulated errors can be quantified or pre-analyzed when originating from a variety of sources--e.g., cumulative errors in a measurement of the length of a line as a result of errors in temperature determination, tension determination, marking the line endpoints, etc.

Redundancy. See degrees of freedom.

Relative accuracy. The calculatable uncertainty, for a given level of probability (e.g., 95%), in the location of any point or corner relative to other points or corners set, found, established, or reestablished.

Residual. The difference between an independent observation or measurement in a set of observations and the mean of those observations.

Significant figures. The number of figures that have meaning in any measured quantity as estimated from the order of precision with which the quantity was measured.

Standard deviation. A statistic indicating the spread of measured values about the mean for a given quantity.

Standard error of the mean. The standard deviation of the probability function or probability density function for a random variable and especially of a statistic, in this case, the mean.

Student's "t" distribution. A sampling distribution used for small samples.

True value. The actual value existing in nature. Due to the existence of random errors in measurement, the true value can usually only be estimated (as the mean of multiple measurements).

Uncertainty. Because all measurements have associated random errors, there is *uncertainty* that any particular measurement represents the true value of the quantity being measured. More commonly called precision, *uncertainty* can be expressed mathematically using standard deviation to determine the range within which (positive or negative random) errors are expected to fall at a given level of probability (e.g., 95%). One might mathematically describe precision as the range within which the true measurement lies with 95% probability or certainty. The terms certainty and uncertainty are commonly used interchangeably in surveying literature.

Table 1

Values of the t Statistic to be Used in Calculating Confidence Intervals.

Degrees of Freedom	50	70	90	95	98	99
	Confidence Level (percent)					
1	1.000	1.963	6.314	12.706	31.821	63.657
2	0.816	1.386	2.920	4.303	6.965	9.925
3	0.765	1.250	2.353	3.182	4.541	5.604
4	0.741	1.190	2.132	2.776	3.747	4.604
5	0.727	1.156	2.015	2.571	3.365	4.032
6	0.718	1.134	1.943	2.447	3.143	3.707
7	0.711	1.119	1.895	2.365	2.998	3.499
8	0.706	1.108	1.860	2.306	2.896	3.355
9	0.703	1.100	1.833	2.262	2.821	3.250
10	0.700	1.093	1.812	2.228	2.764	3.169
11	0.697	1.088	1.796	2.201	2.718	3.106
12	0.695	1.083	1.782	2.179	2.681	3.055
13	0.694	1.079	1.771	2.160	2.650	3.012
14	0.692	1.076	1.761	2.145	2.624	2.977
15	0.691	1.074	1.753	2.131	2.602	2.947
16	0.690	1.071	1.746	2.120	2.583	2.921
17	0.689	1.069	1.740	2.110	2.567	2.898
18	0.688	1.067	1.734	2.101	2.552	2.878
19	0.688	1.066	1.729	2.093	2.539	2.861
20	0.687	1.064	1.725	2.086	2.528	2.845
21	0.686	1.063	1.721	2.080	2.518	2.831
22	0.686	1.061	1.717	2.074	2.508	2.819
23	0.685	1.060	1.714	2.069	2.500	2.807
24	0.685	1.059	1.711	2.064	2.492	2.797
25	0.684	1.058	1.708	2.060	2.485	2.787
26	0.684	1.058	1.706	2.056	2.479	2.779
27	0.684	1.057	1.703	2.052	2.473	2.771
28	0.683	1.056	1.701	2.048	2.467	2.763
29	0.683	1.055	1.699	2.045	2.462	2.756
30	0.683	1.055	1.697	2.042	2.457	2.750
∝	0.674	1.036	1.645	1.960	2.326	2.576

It is unlikely that the surveyor will ever know the true values with total certainty for measured distances, angles, and differences in elevation. However, the observer's willingness to accept a value for the quantity should increase with the number of times a measurement is repeated using consistently good techniques and adjusting for systematic errors. The true value μ is best estimated as $\bar{x}$, the average of the measurements. Each additional measurement is likely to change that best estimate somewhat, although there is a practical limit to the number of times a surveyor can measure any quantity. Nevertheless, an appropriate range of acceptable values will be helpful when professional judgements are made regarding the relationship of a surveyor's measurements to those made by other surveyors.

Just as the best estimate $\bar{x}$ of a surveyed quantity is pragmatically accepted in place of the true value μ, so the standard deviation of a measured quantity,

$\sigma = \sqrt{\frac{(x - \mu)^2}{N}}$, is estimated by,

$s_x = \sqrt{\frac{(x - \bar{x})^2}{n - 1)}}$ to quantify the spread of the measured values, x, obtained in measuring the quantity n times. (The expression for true standard deviation is only correct when the total population containing N values is known.)

If in considering the acceptability of measurements by other surveyors, the surveyor chooses to be right 68.3% of the time (i.e., within one standard deviation), the range of acceptable values would include the best estimate of the mean plus or minus the estimate of the standard deviation of the mean, $\bar{x} \pm \frac{s_x}{\sqrt{n}}$.

Alternately, if the situation required a higher level of certainty, say 95%, the corresponding range of values or the confidence interval reaches from $\bar{x} - (s_{\bar{x}})(t)$ to, $\bar{x} + (s_{\bar{x}})(t)$, where $s_{\bar{x}} = \dfrac{s_x}{\sqrt{n}}$ and t is the tabularized value for the 95% confidence level and the corresponding degrees of freedom n-1. The confidence level is numerically equivalent to the proportion of area under the normal curve which falls between $\bar{x} \pm (s_{\bar{x}})(t)$.

The 90% confidence interval for a measured quantity, $\bar{x} \pm (s_{\bar{x}})(t)$, diminishes to 2.92/6.31 = 46% when the surveyor increases from two measurements (n-1=1), as demonstrated in Figure T1.1, to three measurements (n-1=2) of a quantity, as demonstrated in Figure T1.2. These figures also show that the 95% confidence interval decreases to 4.30/12.71 = 33.8% of its range if the same number of measurements occur. The reader could show a corresponding decrease in range for the 99% confidence interval to 15.6% of its range.

These dramatic changes occur in the range of statistically acceptable values when more replications are made, effectively reducing the error limits. Similarly, the interval decreases to 15.4% of its value when changing from two measurements (one degree of freedom, t=12.71) to a very large number of independent measurements (t=2.04 for thirty degrees of freedom). Application of these calculations through surveying "experience" is described in Section 2.5.

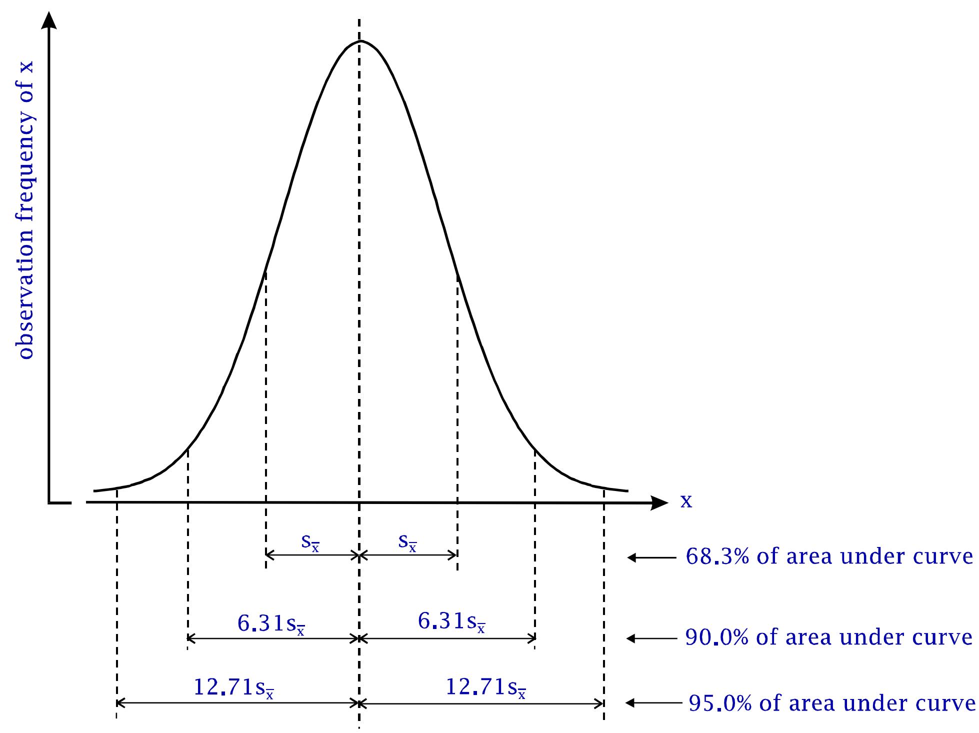

FIGURE T1.1. Probability distribution for two observations, one degree of freedom.

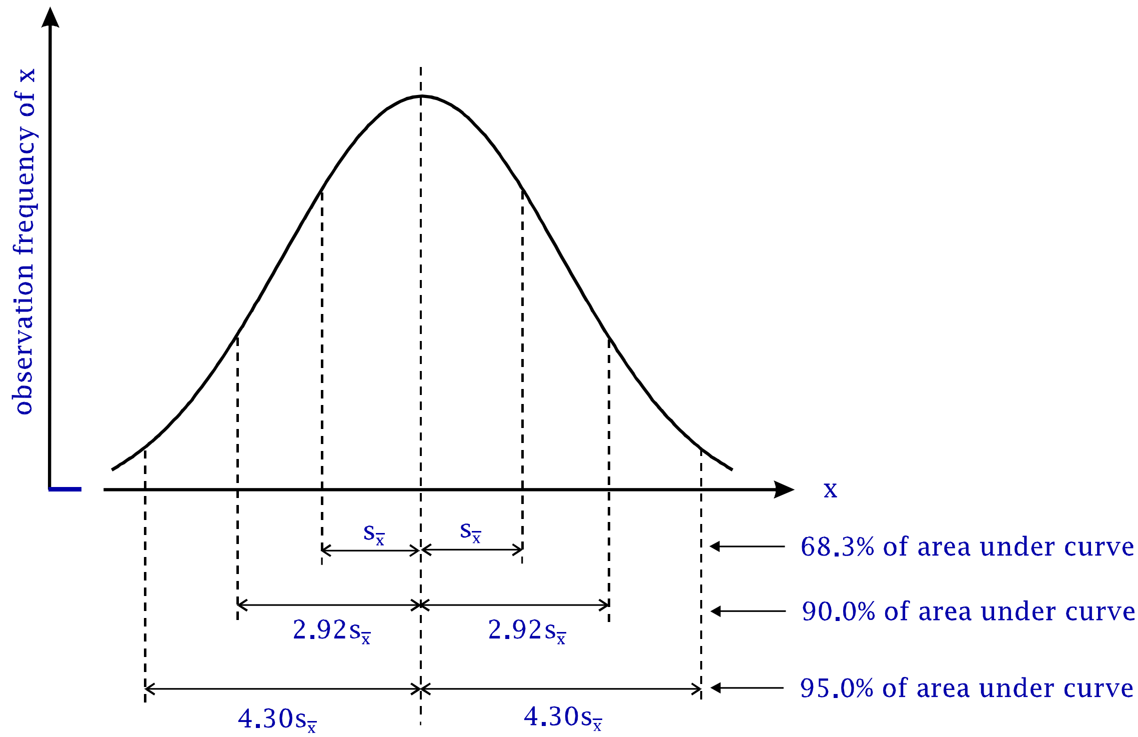

FIGURE T1.2 Probability distribution for three observations, two degrees of freedom.

Table 2

Coefficients for Comparison of Error Ellipse Sizes at Selected Levels of Confidence.

Probability	*c*
0.393	1.000
0.500	1.177
0.632	1.414
0.683	1.516
0.865	2.000
0.900	2.146
0.950	2.447
0.989	3.000
0.990	3.035
0.998	3.500

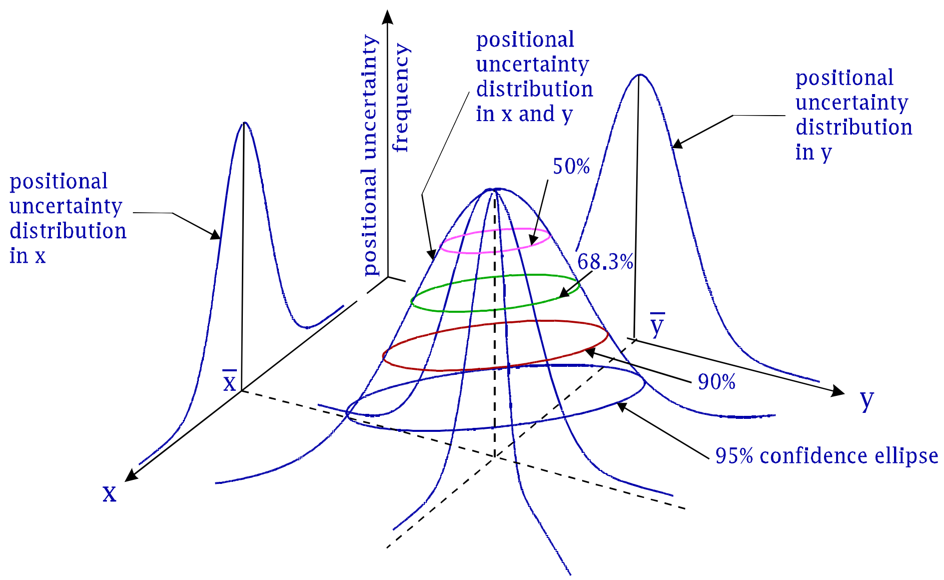

FIGURE T2.1. Calculated position uncertainty in x and y based on angle and distance measurements. (Not to scale)

The range of values describing the measurement of a distance, angle, or difference in elevation between two points can be described by a single probability density function. Likewise, the result of two simultaneously measured entities can be represented by a joint probability density function.

Figure T2.1 demonstrates the result of combining traverse measurements (distances and angles) and related error estimates to determine the probable position of a point in x and y coordinates. For a given probability, the size of the related error ellipse can be described with semimajor and semiminor axes, multiplying the corresponding estimates of standard deviation s_x and s_y by the probability dependent coefficient *c* found in the table.

The joint or bivariate normal distribution presented here describes the region that a point might occupy at the end of a line or series of lines for which the lengths were measured and the directions determined by angular measurement from a base line. Our best estimate $\bar{x}$, $\bar{y}$ of the position of our point is the mean of calculated coordinates from multiple measurements of the distances and angles. The spread in calculated positions is described in the x and y directions by the estimates of standard deviation s_x and s_y for the single normal probability density functions shown as projections above the x and y axes, respectively.

While single and joint probability density functions theoretically include possible values out to infinity, the values of practical significance include a very small region dependent upon the precision with which the distance and angular measurements are made. Based upon arbitrary volume units, the volume under the joint normal distribution is 1.0. If we were to contain 0.95 of that volume above a plane parallel with the x-y axis, the intersection of this plane and the joint normal distribution function would be an ellipse, such as the lowest one in the figure. Correspondingly, lesser percentages of the volume could be isolated by higher planes with correspondingly smaller ellipses. Projecting these ellipses onto the x-y plane results in concentric ellipses, each surrounding a region within which the position of the point might be found with the corresponding level of certainty (see Figure T2.2). The best estimate is the value at the center $\bar{x}$, $\bar{y}$, but the ellipse describes the region within which the true position of the point can be found with increasing levels of certainty.

For further implications, see Section 3.3.

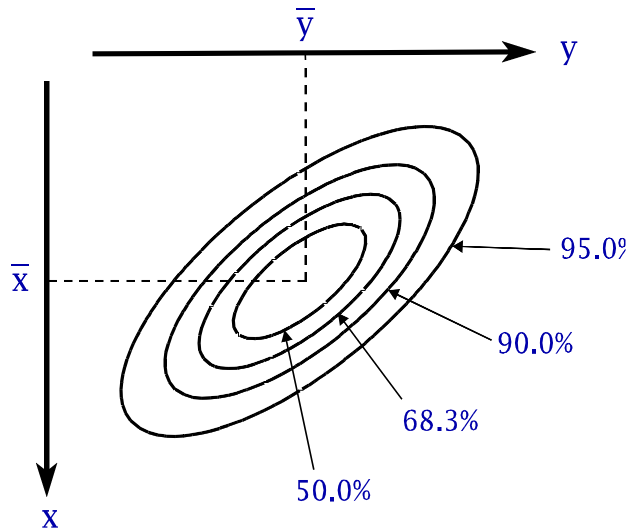

FIGURE T2.2. Error ellipses showing the positional uncertainty in the coordinates of a point.